How precisely can we determine the πNN coupling constant from the isovector GMO sum rule?

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Abstract

The isovector GMO sum rule for zero energy forward πN scattering is critically studied to obtain the charged πNN coupling constant using the precise π^-p and π^-d scattering lengths deduced recently from pionic atom experiments. This direct determination leads to $g_c^2/4\pi = 14.23 \pm 0.09$ (statistic) ± 0.17 (systematic) or $f_c^2/4\pi = 0.0786(11)$. We obtain also accurate values for the πN scattering lengths

INTRODUCTION: ROBUST FORM OF THE GMO RELATION

The analysis to determine the πNN coupling constant should be clear and easily reproducible. One should do a detailed study for the statistical and systematic errors. The precise determination is an absolute statement, it could be erroneous and it should be improvable. In this perspective the Goldberger-Miyazawa-Oehme (GMO) sum rule[1] might be a good candidate. It is a forward dispersion relation at zero energy for πN scattering. It assumes scattering amplitudes to be analytical functions satisfying crossing symmetry. At first isospin symmetry does not have to be assumed and it reads (for more details see e.g.[2]) with its numerical coefficients: $g_c^2/4\pi = -4.50\ J^- + 103.3\ [(a_{\pi^-p} - a_{\pi^+p})/2]$, where J^- is in mb the weighted integral, $J^- = (1/4\pi^2)\int_0^\infty (dk/\sqrt{k^2+m_\pi^2})[\sigma^{Total}_{\pi^-p}(k) - \sigma^{Total}_{\pi^+p}(k)]$ and $a_{\pi^\pm p}$ are the $\pi^\pm p$ scattering lengths in units of m_π^{-1}. All ingredients are physical observables but so far the lack of precision in $a_{\pi^\pm p}$ (contribution of 2/3 to $g^2/4\pi$) led to applications of the GMO relation as consistency check or constraint[3]. The 1s width of the π^-p atom[4] determines $a_{\pi^-p\to\pi^0 n} = -0.128(6)\ m_\pi^{-1}$ and assuming isospin symmetry this gives $a^- = (a_{\pi^-p} - a_{\pi^+p})/2$ and $g_c^2/4\pi = 14.2(4)$ using[5] J^-= -1.077(47) mb. This is not accurate enough although improvements will come[6].

We here report on a possible way to improve the precision on $g_c^2/4\pi$[7]. As a_{π^-p} is precisely known (0.0883(8) m_π^{-1}) from energy shift in pionic hydrogen[8] one can write:

$$g_c^2/4\pi = -4.50\ J^- + 103.3\ a_{\pi^-p} - 103.3\ (\frac{a_{\pi^-p} + a_{\pi^+p}}{2}). \tag{1}$$

and using the above cited J^- (to be calculated later), $g_c^2 = 4.85(22) + 9.12(8) - 103.3[(a_{\pi^-p} + a_{\pi^+p})/2]$. This (not our final result) shows that all the action is in the term $1/2(a_{\pi^-p} + a_{\pi^+p})$,which, assuming isospin symmetry, is a^+. If this quantity is positive $g_c^2/4\pi$ is smaller than 14, if it is negative it is larger. One way to determine the small a^+ is to use the accurate π^-d scattering length $a_{\pi^-d} = -0.0261(5)\ m_\pi^{-1}$, from the pionic deuterium 1s energy level[9]. To leading order this is the coherent sum of the π^- scattering lengths from the proton and neutron, which, assuming charge symmetry (viz, $a_{\pi^+p} = a_{\pi^-n}$) is the term required in our 'robust' relation (1) The strong cancellation between the two terms is then done by the physics. In order to match the precision using the width, we only need a theoretical precision in the description of the deuteron scattering length to about 30%.

ZERO-ENERGY π^--DEUTERON SCATTERING AND a^+

In multiple scattering theory of zero-energy s-wave pion scattering from point-like nucleons and in the fixed scattering-center approximation, the leading contribution is[10]:

$a^{static}_{\pi^- d} = S + D$ ... with $S = [(1+m_\pi/M)/(1+m_\pi/M_d)](a_{\pi^- p}+a_{\pi^- n})$, M and M_d being the nucleon and deuteron masses respectively. The double scattering term D is:

$$D = 2 \frac{(1+m_\pi/M)^2}{(1+m_\pi/M_d)} [(\frac{a_{\pi^- p}+a_{\pi^- n}}{2})^2 - 2(\frac{a_{\pi^- p}-a_{\pi^- n}}{2})^2] < 1/r >, \tag{2}$$

and with our final scattering lengths $D = -0.0256\ m_\pi^{-1}$ quite close to $a_{\pi^- d}$ experimental.

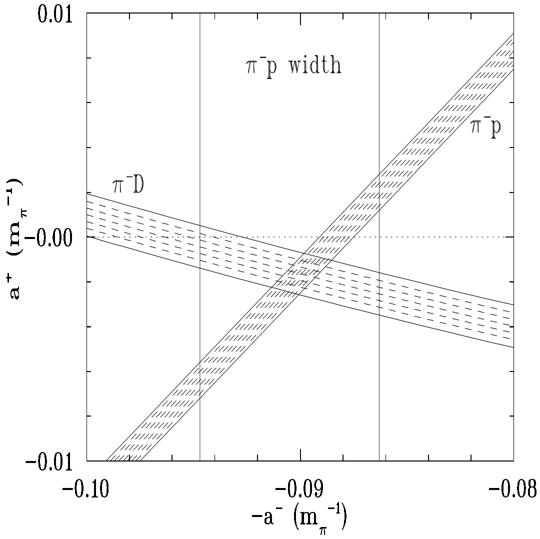

Fig. 1. Our graphical determination of the πN scattering lengths in excellent agreement with the central values of the experimental PSI group[4], $a^+ = -22(43)\cdot 10^{-4} m_\pi^{-1}; a^- = 905(42)\cdot 10^{-4} m_\pi^{-1}$.

We shall here follow the recent theoretical multiple scattering investigation of Baru and Kudryatsev (B-K)[11]. The comparison of typical contributions is listed in Table 1. 1)*Fermi motion*: the nucleons have a momentum distribution which produces an attractive

Table 1. Typical contributions to $a_{\pi d}$ in units of $10^{-4} m_\pi^{-1}$, recall $a^{exp.}_{\pi d} = -261(5)$[9].

Contri -butions	D	Fermi motion 1)	Absorption corr.[16] 2)	s-p interf. 3)	$(\pi^- p, \gamma n)$ double scatt.	Form factor 4)
Present	-256(7)	61(7)	-56(14)	small	-2	17(9)
B-K	-252	50	X	-44	X	29(7)
Contri- butions		Non-static effects 5)	Isospin violation 6)	Higher order 6)	p-wave double scatt. 6)	Virtual pion 6)
Present		10(6)	3.5	4(1)	-3	-7(2)
B-K		10	3.5	6	-3	X

contribution, calculable to leading order from $< p^2 >$ of the nucleon momenta in the deuteron. The uncertainty of 7 comes from the D-state percentage in the deuteron, P_D =4.3% vs. 5.7% for the Machleidt1[12] vs. the Paris[13] wave functions. 2) *Absorption correction*: the absorption reaction π^-d →nn, using 3-body Faddeev approaches[14,15,16] produces a repulsive (−20%) contribution (not included in B-K). These studies were done carefully but a modern reinvestigation of this term is highly desirable. 3) *"s-p' interference*: a −15% correction was obtained by B-K. We find that it is a model dependent contribution due nearly entirely to the Born term the contribution of which vanishes exactly. We have then not considered this contribution. 4)*Form factor*: this non-local effect enters mainly via the dominant isovector πN s-wave interaction, closely linked to ρ exchange. It represents only a correction of about −10%. 5) *Non-static effects*: these produce only a rather small correction of about 4%. There are systematic cancellations between single and double scattering as was first demonstrated by Fäldt[17]. It has been numerically investigated by B-K and we have adopted their value, the error of 6 reflects a lack of independent verification. 6) *Isospin violation, higher order terms, p-wave double scattering,virtual pion scattering*: these corrections are all small and controllable. The isospin violation in the

πN interaction comes in part from the $\pi^{\pm} - \pi^0$ mass difference where an additional check comes from the chiral approach[18]. Based on this, we obtain the preliminary, though nearly final, values $(a_{\pi^- p} + a_{\pi^- n})/2 = (-17 \pm 3(\text{statistic}) \pm 9(\text{systematic}))10^{-4} m_\pi^{-1}$ and $(a_{\pi^- p} - a_{\pi^- n})/2 = (900 \pm 12)\ 10^{-4} m_\pi^{-1}$. Our values represent a substantial improvement in accuracy as seen in Fig. 1. The contribution of the scattering lengths to $g_c^2/4\pi$ has here a precision of about 1%.

TOTAL CROSS SECTION INTEGRAL J^-

The cross section integral contributes only one third to the GMO relation. Total cross sections are inherently accurate and their contribution is calculated with accuracy, but for the high energy region. The possibility of systematic effects in the difference must be considered, particularly since Coulomb corrections have opposite sign for $\pi^{\pm}$p. The only previous evaluation with a detailed discussion of errors is that by Koch[5]. Later evaluations given in Table 2 find values within the errors, but the uncertainties are not stated and analyzed. In view of obtaining a clear picture of the origin of uncertainties we

Table 2. Some values of J^- in mb, Ref.[19] uses 2 different PWA: K-H[20] and their own, VPI.

Ref.	Koch 1985[5]	Workman *et al.* 1992; K-H[19]	Workman *et al.* 1992; VPI[19]	Arndt *et al.* 1995[21]	Gibbs *et al.* 1998[22]	Present work
J^-	-1.077(47)	-1.056	-1.072	-1.05	-1.051	-1.095(31)

have reexamined this problem in spite of the consensus. We limit the discussion to the critical features. The typical shape of the integrands is shown in Fig. 2 up to 2 Gev/c.

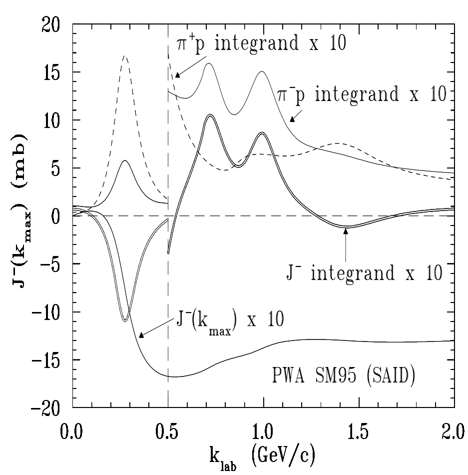

Fig. 2. The J^- separate integrands for $\pi^{\pm}$p as well as their difference as function of k_{lab} together with the cumulative value of the integral $J^-(k_{lab})$ from the region $0 < k < k_{lab}$. The integrands are in units of mb GeV/c.

There are no total cross-section measurements below 160 MeV/c, but the accurately known $a_{\pi^{\pm} p}$ give a strong constraint assuming isospin symmetry. The s- and p-wave contributions nearly cancels. A tiny correction occurs, since isospin is broken by the 3.3 MeV lower threshold for the π^0n channel below the physical π^-p threshold. The main contributions come from the region of the Δ resonance and just above. There are no strong cancellations in the difference between $\pi^{\pm}$p cross sections in that region and the cross sections have been very carefully analyzed. We have first evaluated the hadronic cross sections up to 2 GeV/c based on the VPI phase-shift solution[23]. In doing so Coulomb corrections and penetration factors have been taken into account in the adjustment to experimental data even if the treatment may not be optimal. It also allows for some isospin breaking, since the Δ mass splitting is parameterized[24]. In view of the not so high accuracy we aim for, this should be adequate. Bugg[25,26] has emphasized that in the π^+p scattering the total cross sections are systematically reduced at all energies by the Coulomb repulsion between the particles and, conversely, enhanced in π^-p scattering. One must correct for

this effect, which gives a negative contribution to J^-. Having made no correction for it at higher energies means that we will underestimate the coupling constant somewhat. In the region around 500 MeV/c there are long-standing problems with the experimental total cross section data[24]. This uncertainty, larger than the Coulomb penetrability effects, should be resolved. So we have preferred to use the SM95 PWA solution as the best guide. The real uncertainty in J$^-$ comes from the high energy region and is linked to the relatively slow convergence of the integral.

Table 3. Contributions to J$^-$ in mb according to interval of integration and to the total cross-section input. 'Selected" is for the world data[28] with statistical and systematic errors $\leq 1\%$. Here the first given error is statistical and the second one systematic.

k(GeV/c)	0 to 2	2 to 4.03	4.03 to 240	240 to ∞
Input	SM95[23]	Selected[28]	Selected[28]	Regge(94)[27]
J$^-$(mb)	(-1.302±0.006)(17)	(0.064±0.002)(7)	(0.133±0.005)(24)	(0.030)
k(GeV/c)	0 to 2	240 to ∞	0 to ∞	0 to ∞
Input	Arndt98)[23]	Regge(98)[28]	SM95+Regge94	Arndt98+Regge98
J$^-$(mb)	(-1.329±0.006)(17)	(0.018)(3)	(-1.075±0.008)(30)	(-1.114±0.008)(30)

We have evaluated the different contributions (see Table 3) with no other Coulomb and penetration corrections than those introduced by the experimental authors above 2 GeV or by the theoretical analysis below 2 GeV. We find, based on (integration of hadronic cross section) the SM95 and Arndt 12/98 analysis below 2 GeV/c[23], and on the Regge pole PDG94[27] and PDG98[28] extrapolation beyond 240 GeV/c, the values $J^- = (-1.075 \pm 0.008)(30)$ mb and $(-1.114 \pm 0.008)(30)$ mb respectively. We have adopted the mean value $J^- = (-1.095 \pm 0.008)(30)$ mb. In our calculation we have added a systematic uncertainty from Coulomb penetration effect of ± 0.017 from the region less than 2 GeV/c.

RESULTS AND CONCLUSIONS

We have derived first new values for the πN scattering lengths from the π^-d one:

$$a^+ \simeq \frac{a_{\pi^-p} + a_{\pi^-n}}{2} = (-17 \pm 3)(9)) \cdot 10^{-4} m_\pi^{-1}, a^- \simeq \frac{a_{\pi^-p} - a_{\pi^-n}}{2} = 900(12) \cdot 10^{-4} m_\pi^{-1}.$$

Our second conclusion concerns the charged πNN coupling constant using these new accurate values in (1) with $J^- = (-1.095 \pm 0.008)(30)$ and charge symmetry:

$$g_c^2/4\pi = (4.93 \pm 0.04)(14) + (9.12 \pm 0.08) + (0.18 \pm 0.03)(9) = (14.23 \pm 0.09)(17). \quad (3)$$

The uncertainty comes mainly from J^-. This coupling constant which agrees quite well with the text book value, 14.28(18)[20] is intermediate between the low value deduced from the large data banks of NN and πN scattering data[24,29] and the high value from np charge exchange cross sections[30]. It is fully compatible with the latter, differing statistically by only about one standard deviation.

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